Unlike shock compression data at stresses greater than the HEL, uncertain effects of the stress configuration are minimal and interpretation of the data is more direct. Even though ultrasonic measurements are far more precise than shock compression techniques, the larger compressions of the shock experiment enhance higher order effects sufficiently to be observable.

Fowles[25] has recently demonstrated that Thurston's finite strain theory can be adapted so that shock compression data in the elastic range can be used along with ultrasonic data to determine higher order elastic constants. Fowles[25] developed an explicit stress vs. compression relation involving higher order elastic constants for large uniaxial elastic compression which we will use to interpret the shock compression data. In order to test the adequacy of the usual second order elastic constant description of sapphire at large compressions, the data of Gieske and Barsch are used to calculate the stress-volume relations from Fowles' finite strain development. The results of the calculations are shown in Fig. 3 along with the shock compression data. At the smaller compressions the different crystallographic orientations cannot be distinguished between at the scale of the figure and the differences at the largest compressions are less than the experimental error. Recent shock compression data of Barker *et al.*[37] on 0° sapphire are also included in the figure.

Examination of the extrapolated ultrasonic data in Fig. 3 shows a systematic deviation between the compression computed from second order constants and the observed shock compressions. The value of the third order elastic constant which gives the best fit to the shock data is obtained by determining the difference between the stress compression curve calculated from the second order constants and the observed shock compression data. Within the accuracy of the data, $C_{111} \approx C_{333} = -(3{\cdot}6 \pm 0{\cdot}4) \times 10^{13}$ dyne/cm².

Because third order elastic constants give sufficiently good description to the shock

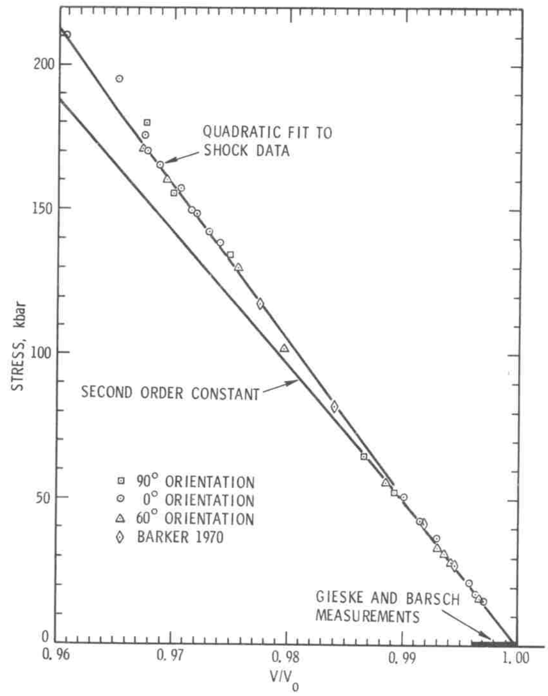

Fig. 3. Shock compression of sapphire in the elastic range. The second-order elastic constants of Gieske and Barsch are used in Fowles' finite strain equations to predict the large strain results in the 0° and 90° orientations indicated by the second order constant line. The slight negative curvature of the second order constant line is a result of finite strain theory. The difference between the shock results, as fit by a quadratic, and the finite strain values predicted from the usual second-order constant predictions allow third order elastic constants to be computed. Recent shock data of Barker are also included. To the scale used in the figure the compressions in the various crystallographic orientations cannot be distinguished between. The compression range of the hydrostatic pressure measurements of Gieske and Barsch are indicated along the abscissa.

data, extrapolation of the ultrasonically measured bulk modulus and its pressure derivative should provide an accurate estimate of the isotropic compression curve for comparison with the shock compression data above the HEL.*

(b) *High pressure response*

The unusually large HEL values observed for sapphire cause large deviations in stress-

*Note added in proof (See p. 2330).

volume relations from the isotropic compressions achieved in hydrostatic experiments. Furthermore the large shear stresses which are a consequence of the large HEL values cause concern that the shear itself may cause irreversible changes in the properties of the solid. Although it is possible to compare shock data directly with hydrostatic data as one method for determination of shear stress offset in the high pressure region, it is advantageous to develop techniques for determining shear strength from the shock data itself, independent of the hydrostatic data. Conclusions based on the study of the shock data alone impose no *a priori* assumptions concerning the equivalence of the shock and hydrostatic compressions and from a pragmatic point of view the conclusions are not susceptible to changes in the static data or uncertainties concerning the equation of state. Accordingly, we will delay comparing shock and static data until the shock data has been examined for evidence of shear stress offset.

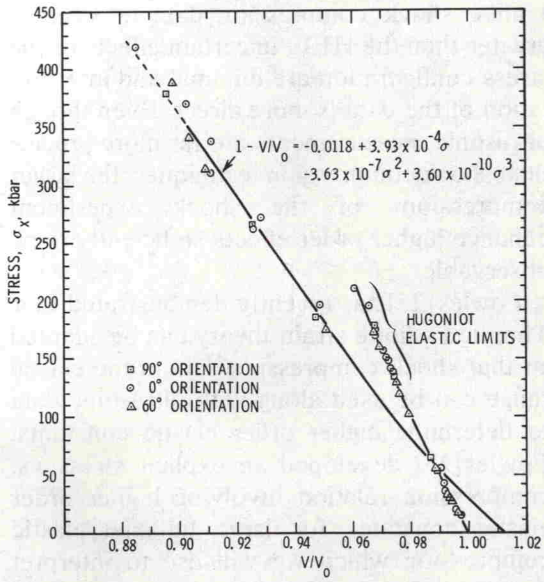

Fig. 4. Stress-volume relations for sapphire under shock-wave compression. The experimental points for the present investigation are fit with the cubic polynomial relation shown. All high pressure points show a common compression curve in spite of the fact that various Hugoniot elastic limits are observed depending upon the crystallographic orientation and driving pressure. For stresses above 350 kbar the data are not as accurately fit by the cubic polynomial.

(c) *Compression observations*

One of the most obvious and striking features of the data shown in Fig. 4 is that all the high pressure compression states fall on a common compression curve. The common compression curve is observed even though HEL values ranging from 120 to 210 kbar were observed for the different crystallographic orientations, sample thicknesses and driving pressures. According to the elastic–plastic model each high pressure experiment should exhibit a shear stress offset equal to the shear strength offset which is proportional to the HEL. Thus, different high pressure compression curves should be observed for different HEL values. The common compression curve indicates the constant shear strength model will not describe the response of sapphire. Although the value of the shear stress offset cannot be obtained from the observation of a common compression curve it is possible to conclude that sapphire experiences a substantial loss of shear strength and a collapse *toward* the isotropic compression curve. The common compression curve, which is an explicit demonstration of a significant loss of shear strength, was also observed for shock-loaded crystalline quartz [24].

(d) *Shock velocity values*

The stress-volume conditions in the immediate vicinity of the HEL are a sensitive indication of changes in the shear strength in the high pressure region. This can be demonstrated by considering equation (5) which relates the longitudinal stress, σ_x, measured in the shock experiment to the mean pressure, $\bar{P}$, and the shear stress offset, σ_τ. The shock compressibility may be obtained from derivatives of equation (5) with volume such that

$$\frac{d\sigma_x}{dV} = \frac{d\bar{P}}{dV} + \frac{d\sigma_\tau}{dV}. \tag{12}$$